Difference vs differential equations : some case stories

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A natural phenomenon can be modelized by differential equations (ordinary or partial) as well as by difference equations or discrete dynamical systems.

For example, Verhulst's population model leads to the ODE

$$p'(t) = p(t)[a - bp(t)] \quad (t \geq 0), \tag{1}$$

where a, b are positive real numbers. The corresponding difference equation is

$$p_{n+1} - p_n = p_n[a - bp_n] \quad (n \in \mathbb{N}). \tag{2}$$

There are similitudes and differences in the solutions of problems (1) and (2), depending upon the values of a and b.

Of course, quantitative closedness, for the same values of a and b, should only been expected between the solutions of (1) and of the difference equation

$$p_{n+1} - p_n = hp_n[a - bp_n] \quad (n \in \mathbb{N}) \quad (n \in \mathbb{N}) \tag{3}$$

when $h > 0$ is sufficiently small. We are far from what is ofter called the discrete Verhulst's model, namely

$$p_{n+1} = p_n[a - bp_n], \quad (n \in \mathbb{N}). \tag{4}$$

This is of course well known, but we will exhibit and analyze, for more general differential equations or systems and their difference counterparts, the presence of similitudes and of differences in the approaches and in the obtained results.